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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 932

THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF THE DRAG OF INSTALLED AIRCRAFT RADIATORS

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Proceedings of the
Fifth International Congress for Applied Mechanics
Cambridge, Massachusetts, September 1938

Washington
February 1940

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THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF THE DRAG OF INSTALLED AIRCRAFT RADIATORS*

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The drag of installed aircraft radiators has been determined in various cases in wind-tunnel and free-flight tests (references 1, 2, and 4). Unfortunately, however, the magnitude of the total drag alone affords no information about the quality of the radiator installation. The present report therefore proposes the determination of the absolute magnitude of the total radiator drag and, in addition, of the different causes of the radiator drag.

In figure 1 is shown a normal radiator installation for a liquid-cooled engine. Now assume the radiator including cowling removed from the nacelle and mounted into a frictionless flat plane exposed to an air current as in figure 2. The contours in direct proximity of the radiator installation shall remain unchanged. The eventual differences in drag and flow volume resulting from the two installation arrangements are to be attributable to the mutual interference between radiator and airplane components. Since this effect is not dealt with here, the analysis can be restricted to the simplified case of installation of figure 2.

Two zones, A and B, are to be recognized in the field of flow of the radiator: zone A to include all streamlines passing through the radiator, zone B all remaining streamlines and up to the plane of intersection a-a passing through the cooling air discharge orifice (fig. 2). The boundary of the two zones forms an area composed of streamlines which partly coincides with the contour of the radiator cowling. Visualizing this surface as a fixed wall, the total drag of the radiator arrangement can be divided into drag W_A and W_B, whereby drag W_A is equal to the force

*"Theoretische und experimentelle untersuchungen ueber den Luftwiderstand eingebauter Flugzeugkuehler." Proceedings of the Fifth International Congress for Applied Mechanics, Cambridge, Massachusetts, September 12-16, 1938, pp. 565-570.

in inflow direction on the boundary of zone A and drag W_B equal to the force on the boundary of zone B.

Notation

v, velocities in the plane of intersection a-a.

v_a, velocity in cooling-air-discharge section.

v_K, velocity in assumedly free radiator section.

v_o, velocity in undisturbed flow.

p, absolute pressures in plane a-a.

p_a, pressure in cooling-air-discharge opening.

F_K, frontal area of radiator element.

W, total drag of installed radiator.

F_a, area of cooling-air-discharge opening.

ρ, air density

$q_o = v_o^2 \frac{\rho}{2}$ dynamic pressure of air stream.

V, cooling-air volume in time unit.

Δp_K, pressure gradient in radiator element.

Subscripts x, y, z, denote the components of the velocity in the coordinate directions x, y, z.

If assuming the flow devoid of any source or sink, the drag W_A and W_B follows from the impulse theorem at

$$W_B = \iint^{B} -p + \rho\, v_x(v_o - v_x)\, dF \tag{1}$$

$$W_A = \iint^{A} -p + \rho\, v_x(v_o - v_x)\, dF \tag{2}$$

$$W = W_A + W_B = c_w \frac{\rho}{2} v_o^2 F_K \tag{3}$$

where dF designates a surface element, and the integration extends over the zone A or B of plane a-a. When

$$g = p + \frac{\rho}{2}\left(v_x{}^2 + v_y{}^2 + v_z{}^2\right) = \frac{\rho}{2} v_0{}^2 - \Delta g \tag{4}$$

where g is the total pressure and Δg, the energy loss of a stream filament up to plane a-a relative to a stream filament at infinity W_A can equally be written as

$$W_B = \frac{\rho}{2}\iint_A^b \left[-(v_0 - v_x)^2 + v_y{}^2 + v_z{}^2\right] dF + \iint_A^B \Delta g \; dF \tag{5}$$

$$W_A = \frac{\rho}{2}\iint \left[-(v_0 - v_x)^2 + v_y{}^2 + v_z{}^2\right] dF + \iint \Delta g \; dF \tag{6}$$

Now the assumption is made that the pressure and the speed in the cooling-air-discharge opening can be expressed by a mean value and furthermore, that the velocity components along the y, z axis in the cooling-air-discharge opening are negligibly small. Then the relation for W_A becomes

$$W_A = \frac{\rho}{2}\left[\Delta g_a - \frac{\rho}{2}(v_0 - v_a)^2\right] F_a \tag{7}$$

Δg_a indicates the energy loss on passing through the radiator and amounts to

$$\Delta g_a = - p_a - \frac{\rho}{2} v_a{}^2 + \frac{\rho}{2} v_0{}^2 \tag{8}$$

Thus with given flow volume and known pressure in the cooling-air-discharge opening the drag W_A can be computed.

As concerns the magnitude of W_B, the following prediction can be made; with aerodynamically correct radiator cowling, the energy losses in circulation about the radiator will be quite low, so that the second term of equation (5) is, in this case, negligible. The magnitude of the first term can be estimated as follows; Visualize the radiator replaced by a certain source and sink distribution in zone A, which is so arranged that the boundary surface of zones A and B becomes a stream surface. The force exerted internally on the stream surface must, in this case, be equal to the pressure exerted on the stream surface from the outside, i.e., in this case

$$W_B = - W_A$$

Assuming that the speed in the discharge opening is replaceable by the average value v_a^* and corresponds to the pressure p_a by loss-free flow, the yield Q of the source-sink distribution must assume the following value,

$$Q = (v_a^* - v_a)\ F_a \tag{9}$$

where

$$\frac{\rho}{2} v_a^{*2} = \frac{\rho}{2} v_o^2 - p_a \tag{10}$$

The momentum theorem applied to zone A gives

$$W_B + \rho\ v_o\ Q = \rho\ v_a^{*2}\ F_a - \rho\ v_a\ v_o\ F_a + p_a\ F_a \tag{11}$$

$$W_B = \frac{\rho}{2} (v_o - v_a^*)^2\ F_a \tag{12}$$

$$W_B = \frac{\rho}{2} v_o^2\ F_a \left(1 - \sqrt{1 - \frac{p_a}{q_o}}\right)^2 = K \tag{13}$$

Incidental to the application of the momentum theory, the propulsion of magnitude $\rho\ v_o\ Q$ exerted on the sources and sinks must, of course, be also taken into account.

Admittedly there might be cases where the above assumptions are no longer permissible and other sources and sinks outside of zone A must be assumed. In such cases the expression equation (13) assumes, as a rule, smaller or even negative values, but not exceeding the amount according to equation (13). (Compare, for instance, the assumptions according to (3), (5).) Since the pressure p_a nearly always ranges between 0 and $\pm 0.5\ q_o$, the expression according to equation (13) always remains small relative to W_A and can therefore be disregarded.

With equations (3), (7), (8), and (13), and given dimensions, known cooling volume and known pressure in the cooling-air-outlet opening, the air resistance of the radiator assembly can be predicted. This resistance computed on the assumption of loss-free flow outside the radiator cowling is, in contradistinction from the measured total drag W, designated with W^*. If there is a difference between W and W^*, it must be due to energy losses outside the radiator, i.e., the second term of equation (5) will not be equal to zero. Therefore:

$$W = W^* + \iint^{B} g \, dF = c_w \frac{\rho}{2} v_o^2 F_K \qquad (14)$$

$$W^* = c_w^* \frac{\rho}{2} v_o^2 F_K = W_A \pm K \qquad (15)$$

In practice, of course, the avoidance of energy losses outside the radiator cowling, say, by greater smoothness of the cowling, etc., will make the changes in v_a and p_a quite small, so that the difference of W and W^* does not exactly correspond to the actually obtainable gain in drag. However, this effect is very small and disregarded for reasons of simplicity.

The radiator drag is chiefly due to the energy losses on flowing through the radiator. To combat the power input, given by cooling-air volume V and the pressure gradient Δp_K in the radiator element, a certain minimum drag

$$W_m = V \Delta p_K \frac{1}{v_o} = \Delta p_K F_K \frac{c_K}{v_o} = c_{w_m} \frac{\rho}{2} v_o^2 F_K \qquad (16)$$

must be taken in the bargain.

But in reality, the energy loss Δg_a at passage is much higher than corresponds to the pressure gradient Δp_K in the radiator element. Figured with this value, it affords a drag W_i:

$$W_i = V \Delta g_a \frac{1}{v_o} = \Delta g_a F_K \frac{c_K}{v_o} = c_{w_i} \frac{\rho}{2} v_o^2 F_K \qquad (17)$$

The differences between W_m and W_i are due to flow losses on passage through the radiator cowling, the differences between W_i and the computed W^* due to the basic disposition of the radiator. The determination of W^*, W_m, W_i, and W afford an insight into the composition of the radiator drag.

W, W^*, W_m, W_i can equally be appraised mathematically if based on the following simplifying assumptions. Limited to a two-dimensional problem the flow past the radiator is visualized as being bounded by a flat wall, as in figure 4, arranged parallel to the plane of installation. The size of the tunnel section F is put in a certain ratio to the radiator dimensions and so determined that the conditions at the cooling-air outlet agree as much as possible with actual conditions. Since section F does not exceed a certain size, the velocity outside of the cooling-air-outlet section in the plane a-a can be equally replaced by an average value v_i. The pressure in this section is assumed constant and must be equal to p_a. From the equation of continuity follows:

$$F_a\, v_a = F_K\, v_K$$

$$(v_i - v_o)(F - F_a) = F_a\,(v_o - v_a) \tag{18}$$

$$v_i = \frac{F}{F - F_a}\, v_o - \frac{F}{F - F_a}\, v_K$$

Assuming zero pressure in the plane e-e directly before the radiator, and designating with Δg_a the energy losses at passage, with Δg_i the energy losses at flow around the radiator, from plane e-e to plane a-a, we have, according to equation (4):

$$p_a = q_o - \frac{\rho}{2}\, v_a^2 - \Delta g_a \tag{19}$$

$$p_a = q_o - \frac{\rho}{2}\, v_i^2 - \Delta g_i \tag{20}$$

Once the flow losses Δg_i and Δg_a have been ascertained, the cooling volume and the pressure in the outlet opening can be computed with equations (18), (19), and (20). Δg_a consists of the pressure loss Δp_K in the actual radiator element and the loss at inflow and outflow.

For Δg_a and Δg_i, the following appropriate assumptions are made:

$$\Delta g_i = \xi_i\, \frac{\rho}{2}\, v_o^2 \tag{21}$$

$$\Delta g_a = \Delta p_K + \xi_a\, \frac{\rho}{2}\, v_o^2 \tag{22}$$

$$\Delta p_K = c_{w_K} \frac{\rho}{2} v_o^2 \tag{23}$$

ξ_i, ξ_a, and c_{w_K} are constants. The admissibility of the simplifications effected through equations (21), (22), and (23) must be checked in the individual cases, or else properly modified relations introduced for Δg_i, Δg_a, and Δp_K. In the following calculations, the nondimensional quantities, $\frac{c_K}{v_o}$, $\frac{p_a}{q_o}$, $\frac{\Delta g_a}{q_o}$, $\frac{\Delta g_i}{q_o}$, and the nondimensional coefficients, $c_w{}^*$, c_w, c_{w_i}, c_{w_m}, conformably to equations (3), (15), (16), and (17) are employed.

The application of equations (15) and (17) to the present case gives:

$$W = \Delta g_a F_a + \Delta g_i (F - F_a) - \frac{\rho}{2}(v_o - v_a)^2 F_a - \frac{\rho}{2}(v_o - v_i)^2 (F - F_a) \tag{24}$$

$$c_w = \frac{\Delta g_a}{q_o}\frac{F_a}{F_K} + \frac{\Delta g_i}{q_o}\left(\frac{F - F_a}{F_K}\right) - \left(1 - \frac{v_a}{v_o}\right)^2 \frac{F_a}{F_K} - \left(1 - \frac{v_i}{v_o}\right)^2 \left(\frac{F - F_a}{F_K}\right) \tag{25}$$

$$c_w{}^* = \frac{\Delta g_a}{q_o}\frac{F_a}{F_K} - \left(1 - \frac{v_a}{v_o}\right)^2 \frac{F_a}{F_K} - \left(1 - \frac{v_i}{v_o}\right)^2 \left(\frac{F - F_a}{F}\right) \tag{26}$$

$$c_{w_i} = \frac{c_K}{v_o}\frac{\Delta g_a}{q_o} \tag{27}$$

$$c_{w_m} = \frac{c_K}{v_o}\frac{\Delta p_K}{q_o} \tag{28}$$

with the quantities, $\frac{\Delta g_a}{q_o}$, $\frac{v_a}{v_o}$, $\frac{v_i}{v_o}$, $\frac{\Delta g_i}{q_o}$, to be determined according to equations (19) to (23).

Now the derivated equations are applied to an illustrative example. The selection of the constants and the geometric dimensions is as follows:

$$\frac{F_K}{F} = 0.32, \qquad c_{w_K} = 6$$

$$\xi_i = 0.03 \qquad \xi_a = 0.5$$

A flap fitted at the cooling-air outlet (fig. 4) affords different cooling-air-outlet sections F_a. At first, the flow volume $\frac{c_K}{v_o}$ for each flap setting is determined; this is best done graphically, the value of $1 - \frac{p_a}{q_o}$, for each flap setting being plotted against $\frac{c_K}{v_o}$ and according to equations (19) and (20). The intersection of the two curves gives the desired flow volume v_K/v_o.

In this manner, the flow volume $\frac{v_K}{v_o}$ of figure 4 was determined for flap setting $\frac{F_a}{F_K} = 0$, 0.25, 0.50, 0.75, and the related values $\frac{\Delta g_a}{q_o}$, $\frac{\Delta g_i}{q_o}$, $\frac{v_i}{v_o}$, $\frac{\Delta p_K}{q_o}$ according to equations (18), (19), and (20) ascertained, and then the values of c_w, $c_w{}^*$, c_{w_m} and c_{w_i} computed according to equations (25) to (28) and connected by curves. The effect of any changes, as, say in the geometric dimensions, the assumptions regarding flow losses, etc., on drag, flow volume and pressure in the cooling-air outlet are readily detected by this method of calculation.

In the following, the arguments are applied to several experimental radiator investigations. The radiator installation of figure 6 with nacelle and a piece of the wing was explored at full scale in the wind tunnel. Dimensions and arrangement of radiator installation are in the principal points those used in the other example. The radiator assembly was studied at different flap settings and drag, flow volume, and cooling-air-outlet pressure ascertained. The radiator drag followed as difference from the recorded drag less the drag by installed radiator and best nacelle covering. The results are tabulated in table I.

The drag coefficients c_{w_m}, c_{w_i}, $c_w{}^*$ were determined with the help of the foregoing arguments and plotted in figures 7 and 7a. With arrangement 1, a large portion of the drag was found to be caused by the flow losses while circulating around the radiator. Then the radiator inlet was shortened and the inlet edge rounded off, which gave arrangement 2, on which these losses, especially at small flap settings, could be materially lowered, although it did not affect the losses at passage through the radiator, according to figure 7. An intermediate baffle fitted at the radiator inlet conformably to arrangement 3, figure 7a afforded considerable improvement in c_{w_1} and hence indirectly in the total drag. This example indicates how, with the aid of this method, the individual causes of the radiator drag can be ascertained. A similar investigation at smaller scale had been made earlier for a different radiator installation.

A comparison of the computed curves of figure 5 and the measured curves of figure 7 indicates very close agreement, which, considering the extent of necessary simplifications is surprising. The assumption regarding the external flow losses is naturally only conditionally correct, since, with flap closed, these losses must undoubtedly be substantially greater than with flap open, a fact substantiated by experiment. The same also holds for flow losses through the radiator cowling.

By means of the foregoing arguments the problem of radiator drag reduces to the solution of a series of partial problems each of which can be dealt with separately. It affords a clear picture of the composition of the radiator drag and a predetermination of the radiator drag and cooling capacity in most cases with sufficient accuracy.

The employed method is equally applicable to air-cooled engines.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCES

1. Linke, W.: Windkanalmessungen an Kühlerverkleidungen. Jahrbuch 1937 der Deutschen Luftfahrtforschung, vol. 2, p. 165.

2. Muttray, H.: Widerstand und Kühlwirkung eines Flugzeugrumpfes mit verschieden angeordnetem Kühler. Z.F.M., vol. 22, no. 3, Feb. 14, 1931, pp. 65-71; and vol. 24, no. 4, Feb. 28, 1933, pp. 99-103.

3. Barth, W.: Verfahren zur Bestimmung von Grösse und Ursache des Widerstandes eingebauter Flugzeugkühler. Z.f.a.M.M., Dec. 1937, vol. 17, pp. 353-356.

4. Barth, W.: Luftwiderstand und Kühlwirkung von Flugzeugkühlern. Jahrbuch der Deutschen Luftfahrtforschung, 1937, vol. II, pp. 175-181.

5. Barth, W.: Die Bestimmung des Widerstandes und der Durchflussmenge von Kühlern bei verschiedenen Einbauanordnungen. Luftfahrtforschung, vol. 14, no. 6, June 20, 1937, pp. 300-03.

Table I

Test Results with Radiator Arrangement of Figure 4

	$\frac{F_a}{F_K}$	$\frac{v_K}{v_o}$	$\frac{v_a}{v_o}$	$\frac{p_a}{q_o}$	$\frac{\Delta g_a}{q_o}$	$\frac{\Delta p_K}{q_o}$	c_w	c_w* (K=0)	c_{w_m}	c_{w_i}
	0.000						0.320			
	.148	0.104	0.700	-0.06	0.57	0.119	.234	0.071	0.059	0.0123
1	.281	.182	.650	-.04	.618	.222	.234	.139	.113	.0403
	.529	.306	.580	-.235	.899	.537	.401	.382	.275	.1643
	.750	.355	.473	-.383	1.161	.687	.740	.662	.412	.244
	0.000						0.162			
	.148	0.100	0.675	+0.01	0.555	0.113	.106	0.066	0.055	0.0113
2	.281	.179	.636	-.03	.626	.214	.145	.139	.112	.0383
	.515	.314	.608	-.24	.871	.563	.380	.369	.274	.1767
	.750	.390	.520	-.41	1.139	.815	.662	.681	.444	.318
	0.000						0.179			
	.148	0.104					.086			
3	.284	.212	0.747	-0.03	0.475	0.280	.126	0.116	0.100	0.0594
	.750	.370	.493	-.395	1.152	.739	.665	.672	.426	.2734
	.529	.306					.360			

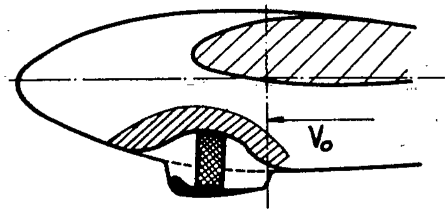


Figure 1.- General arrangement of a radiator assembly for a liquid-cooled engine, mounted in the engine nacelle of an airplane.

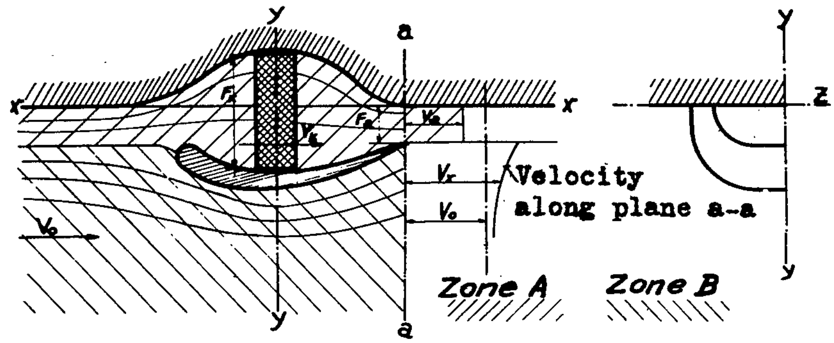


Figure 2.- Radiator installation in infinitely extended frictionless plane.

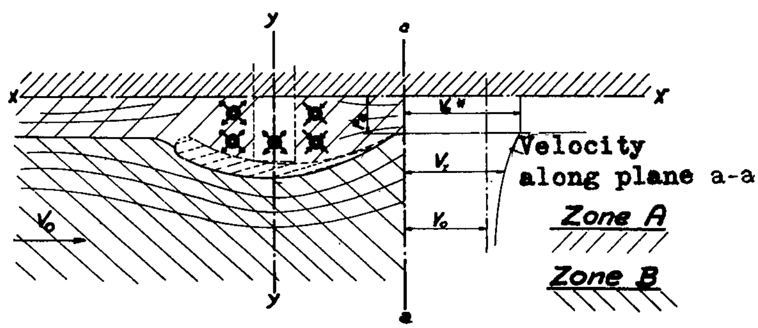


Figure 3.- A specific source and sink distribution substituting for the radiator.

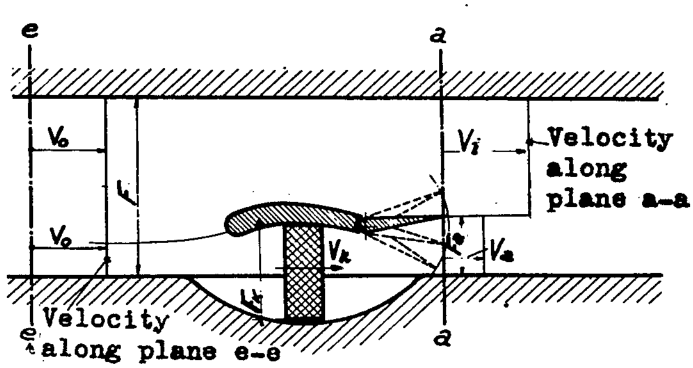


Figure 4.- Radiator in a closed channel

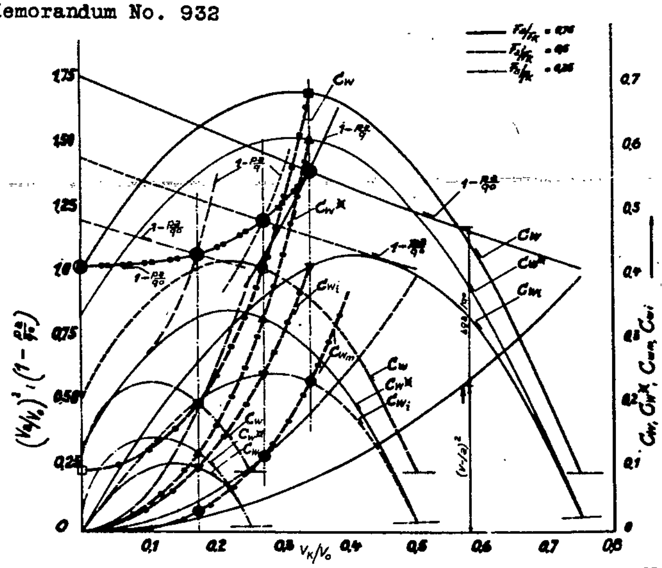


Figure 5.- Theoretical determination of radiator drag and flow volume.

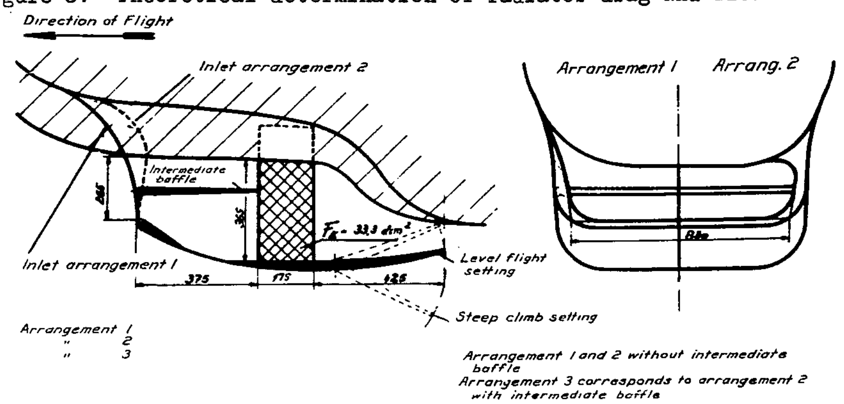
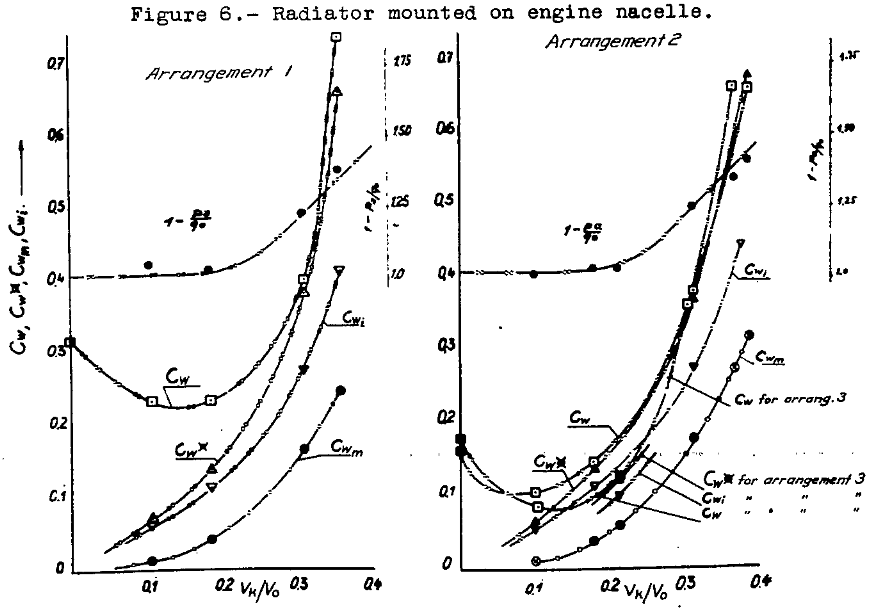


Figure 6.- Radiator mounted on engine nacelle.



Figures 7 and 7a.- Experimental radiator drag and flow volume.